647510 **BSCalgCC5010** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
February/March. -2020
MATHEMATICAL ANALYSIS-1 AND ABSTRACT ALGEBRA-1(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Question-1(A) Attempt any two out of three. **[10]**

(1) X is metric space and $E \subset X$. prove that E is open iff X-E is closed.

(2) X is metric space. Show that union of finite numbers of closed subsets of X is closed. Explain with example that this statement is not true for infinite numbers of closed subsets of X.

(3) Explain cantor set and define it. Show that cantor set is closed.

(B) Attempt any one out of two. **[4]**

(1) d is define on R as; for $\forall$ x,y$\in$R

$d(x,y)=0 \quad ; \quad x=y$

$\quad =\sqrt{x^2+y^2} \quad ; \quad x\neq y$ Then show that (R,d) is metric space.

(2)(i) R is discrete metric space. Find N(5,0.2) in R.

(ii) find E^0 where $E=\{\frac{1}{n} / n\in N\}$.

(iii) construct set of real numbers which has exactly three limit points.

Question-2(A) Attempt any two out of three. **[10]**

(1) prove that any bounded function f defined on [a,b] is R-integrable iff For $\forall\ \varepsilon>0$ there exist partition P of [a,b] such that U(P,f) - L(P,f) < ε , Where $\|P\|<\delta$.

(2) if functions f and g are R-integrable in [a,b] then show that f+g is also R-integrable in [a,b] and $\int_a^b [f(x)+g(x)]dx=\int_a^b f(x)dx+\int_a^b g(x)dx$

(3) using darbour's theorem find value of

$$\lim_{n\to\infty}\frac{1}{n}\left[\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\ldots\ldots\ldots\ldots+\left(\frac{n}{n}\right)^2\right]$$

(B) Attempt any one out of two. **[4]**

(1) Find $\int_1^2 (3x+1)dx$ using definition of R-integration.

(2) if bounded function f on [a,b] is monotonic then show that f is R-integrable.

Question-3(A) Attempt any two out of three. **[10]**

(1) if bounded function f is R-integrable in [a,b] and $F(x)=\int_a^x f(t)dt$,

Where $a\leq x\leq b$ then show that F(x) is continuous in [a,b].

(2) state and prove second mean value theorem of integral calculus.

(3) G=R-{-1}. Binary operation * is defined on G as ; for a,b$\in$G ,a*b=a+b+ab then show

that (G,*) is group and find 2^{-1} in G.

(B) Attempt any one out of two. **[4]**

(1) show that $\int_0^{\frac{\pi}{4}} x\tan(x)dx < \frac{\pi}{8}\log_e 2$.

(2) binary operation * is associative in non empty set A. a,b $\in$ A are non singular elements of A. prove that a*b is also non singular element of A.

Question-4(A) Attempt any two out of three. **[10]**

(1)State and prove lagranges theorem.

(2) Prove that order of permutation f $\in S_n$ is L.C.M. of length of disjoint cycles of f.

(3) G is group and H≤G. then prove that any two left coset of H in G is either equal or disjoint.

(B) Attempt any one out of two. **[4]**

(1) Prove that H={x$\in$G/xa=ax} is subgroup of group G. where a is invariant element of G.

(2) $\sigma \epsilon$ S_6 where $\sigma = \begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 3\,4\,2\,1\,6\,5 \end{pmatrix}$ then find σ^{-1} and o(σ). Also check whether σ is even or odd.

Question-5(A) Attempt any two out of three. **[10]**

(1) if (G,*) ≅ (G',*'). Show that G is commutative iff G' is commutative. Where G and G' are groups.

(2) G is finite group and a ϵ G. prove that O(a)/O(G).

(3) find numbers of generators of cyclic group Z_{pq}. Where p and q are prime Numbers.

(B) Attempt any one out of two. **[4]**

(1) G is group and H≤G. if index of H in G is 2 then prove that H is normal Subgroup of G.

(2) prepare group table in usual notation for quotient group S_3/A_3.

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